

Hyperbolic mask function

The hyperbolic mask denoted $\mathcal{M}(x)$ is a function that transitions smoothly between 0 and 1. It is mostly used to define some regions of the simulation box where specific operators become active. It is a hyperbolic tangent function that takes a value of one inside a specific domain, and vanishes outside. This mask can have two different forms. The first one is such that the mask equals one inside the domain $[x_\ell, x_r]$ and zero outside as

$$\mathcal{M}(x) = \frac{1}{2} \left\{ \tanh \left(\frac{x - x_\ell}{d} \right) - \tanh \left(\frac{x - x_r}{d} \right) \right\}. \quad (1)$$

The d parameter controls the steepness of the mask transition. The second possibility is to have a mask that is equal to zero inside the $[x_\ell, x_r]$ domain, and zero outside. In this case it is expressed as

$$\mathcal{M}(x) = 1 - \frac{1}{2} \left\{ \tanh \left(\frac{x - x_\ell}{d} \right) - \tanh \left(\frac{x - x_r}{d} \right) \right\}. \quad (2)$$

Lastly, it is possible to use a normalized version of these masks function so that their integral is equal to one. For instance Eq. (1) becomes

$$\mathcal{M}(x) = \frac{1}{2V_{\mathcal{M}}} \left\{ \tanh \left(\frac{x - x_\ell}{d} \right) - \tanh \left(\frac{x - x_r}{d} \right) \right\},$$

with $V_{\mathcal{M}} = \int_0^{L_x} dx \mathcal{M}$, and L_x is the total length of the simulation box.