Kinetic source

The source term developed hereafter is inspired by the one currently implemented in GYSELA [1, Appendix A]. It allows to inject independently density and energy. It has the following normalized expression

$$S_{\mathrm{sc}}(x, v_a) = s_{\mathrm{k}} \frac{\mathcal{M}_{\mathrm{sc}}(x)}{\int_0^{L_x} \mathcal{M}_{\mathrm{sc}}(x)\, dx} S_{\mathrm{v}}(v_a) \tag{1}$$

The normalized velocity variable for species a $v_a = v/v_{\mathrm{th}_a}$ is simply denoted by v in the following. The mask function $\mathcal{M}_{\mathrm{sc}}$ defines the spatial extent of the source, most of the time it has a hyperbolic tangent expression, see the `mask_tanh` documentation. The S_{v} term is written as

$$S_{\mathrm{v}}(v) = \left\{ s_0 \left(\frac{3}{2} - \frac{v^2}{2T_{\mathrm{sc}}} \right) + s_2 \left(-\frac{1}{2} + \frac{v^2}{2T_{\mathrm{sc}}} \right) \right\} \frac{1}{\sqrt{2\pi T_{\mathrm{sc}}}} e^{-\dfrac{v^2}{2T_{\mathrm{sc}}}} \tag{2}$$

In Eq. 1 the L_x term stands for the simulation box length. The T_{sc} parameter defines the source temperature, which is constant in space and time. The s_0, s_2 and s_{k} are numerical inputs of the code that allows to define the properties of the source. In general we use $s_0 = 1$ so that the magnitude of the source is controlled by the parameter s_{k} alone. In this case we have indeed $\int dx \int dv\, S_{\mathrm{sc}} = s_{\mathrm{k}}$. Using $s_0 = s_2 = 1$ we recover a Maxwellian source, i.e. in this case

$$S_{\mathrm{v}}(v) = \frac{1}{\sqrt{2\pi T_{\mathrm{sc}}}} e^{-\dfrac{v^2}{2T_{\mathrm{sc}}}} \tag{3}$$

Conversely, by taking $s_2 = 0$ the expression of S_{v} has a vanishing first moment, that is to say in this particular case $\int dv\, S_{\mathrm{v}} = 0$, the source injects energy but no particles. Whatever the values given to s_0 and s_2 the source is symmetric with respect to $v = 0$ therefore it does not inject any net momentum.

1 Derivation of the source

Let $S_{\mathrm{v}}(v)$ be a general source term. We decompose it on the Hermite polynomials basis. By doing this it becomes possible to adjust independently the amount of particles, momentum and energy that this source injects. For two functions f and g let us first introduce the scalar product

$$\langle f, g \rangle = \int_{-\infty}^{+\infty} f(y) g(y) e^{-y^2} dy \tag{4}$$

The first three Hermite polynomials are written as

$$\begin{aligned} H_0 &= 1 & |H_0|^2 &= \sqrt{\pi} \\ H_1 &= 2X & |H_1|^2 &= 2\sqrt{\pi} \\ H_2 &= -2 + 4X^2 & |H_2|^2 &= 8\sqrt{\pi} \end{aligned}$$

The Hermite polynomials form an orthogonal basis for the scalar product defined above, i.e. $\langle H_h, H'_h \rangle = \delta_{h,h'} |H_h|^2$. $\delta_{h,h'}$ is the Kronecker symbol that verifies $\delta_{h,h'} = 1$ if $h = h'$ and $\delta_{h,h'} = 0$ otherwise. Projecting the source term S_v on this basis gives

$$S_v(v) = \sum_{h=0}^{+\infty} c_h \, H_h \left(\frac{v}{\sqrt{2T_\mathrm{sc}}} \right) e^{-\dfrac{v^2}{2T_\mathrm{sc}}} \tag{5}$$

The c_h terms are real valued coefficients. We also introduced here the source temperature T_sc. We now take first three moments of this expansion to retrieve the particle, momentum and energy fluid sources associated with the S_v source term. Using the orthogonality of the Hermite basis we can show that these fluid sources are expressed as

$$\begin{aligned}
\int_{-\infty}^{+\infty} dv \, S_\mathrm{v}(v) &= \sqrt{2T_\mathrm{sc}} \sum_h \langle H_0, c_h H_h \rangle = \sqrt{2\pi T_\mathrm{sc}} \, c_0 \\
\int_{-\infty}^{+\infty} dv \, v S_\mathrm{v}(v) &= T_\mathrm{sc} \sum_h \langle H_1, c_h H_h \rangle = 2\sqrt{\pi} T_\mathrm{sc} \, c_1 \\
\int_{-\infty}^{+\infty} dv \, \frac{1}{2} v^2 S_\mathrm{v}(v) &= \sqrt{2\pi} T_\mathrm{sc}^{3/2} \left(2c_2 + \frac{1}{2} c_0 \right)
\end{aligned}$$

Neglecting in the source expansion Eq. 5 all the terms for $h > 2$ we can obtain a source $\mathcal{S}_n$ that injects particles but no energy nor momentum by setting $c_1 = 0$ and $c_2 = -\frac{1}{4} c_0$. This source can be written as

$$\mathcal{S}_\mathrm{n}(v) = c_0 \left(\frac{3}{2} - \frac{v^2}{2T_\mathrm{sc}} \right) e^{-\dfrac{v^2}{2T_\mathrm{sc}}} \tag{6}$$

Conversely, a source that injects only momentum but no net particles nor energy is defined by $c_0 = 0$ and $c_2 = 0$. It is expressed as

$$\mathcal{S}_\mathrm{u}(v) = c_1 \sqrt{\frac{2}{T_\mathrm{sc}}} e^{-\dfrac{v^2}{2T_\mathrm{sc}}} \tag{7}$$

Lastly, a source that injects only energy can be obtained with $c_0 = 0$ and $c_1 = 0$. It is written as

$$\mathcal{S}_\mathrm{h}(v) = 2c_2 \left(-1 + \frac{v^2}{T_\mathrm{sc}} \right) e^{-\dfrac{v^2}{2T_\mathrm{sc}}} \tag{8}$$

We construct our source S_v using the three independent sources above as $S_\mathrm{v} = \mathcal{S}_\mathrm{n} + \mathcal{S}_\mathrm{u} + \mathcal{S}_\mathrm{h}$. By tuning the c_0, c_1 and c_2 parameter we can inject independently particles, momentum and energy in the plasma. In practice we always take $c_1 = 0$, i.e. the source does not inject any net momentum. Additionally by using $s_0 = \sqrt{2\pi T_\mathrm{sc}} c_0$ and $s_2 = \sqrt{2\pi T_\mathrm{sc}} c_2$ as input parameters in the code we are left with Eq. 2 for the expression of S_v.

References

[1] Y. Sarazin et al. "Predictions on heat transport and plasma rotation from global gyrokinetic simulations". In: *Nuclear Fusion* 51.10 (Sept. 2011), p. 103023. DOI: 10.1088/0029-5515/51/10/103023. URL: https://doi.org/10.1088/0029-5515/51/10/103023.